Low frequency sound propagation in a coaxial cylindrical duct: application to sudden area expansions and to lined expansion chambers

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SUMMARY

A method based on the approximation of the radial pressure profile is developed to analyse the acoustic performance of an expansion and of an expansion chamber at low frequencies. This model is able to predict very accurately the added length of an expansion when there is neither porous material nor perforated tube and can be applied as well when a bulk-reacting lining and perforated tube are included. This model of a dissipative silencer gives results which compare very favorably with experimental data.

1. INTRODUCTION

The cylindrical dissipative silencer is one of the most commonly used devices in practical flow duct acoustic. Its acoustic performance can be predicted in the general case (with flow, with arbitrary external shape, ...) by an FEM approach [1] or by mode matching techniques [2], [3]. However, those methods require a considerable numerical effort which limits their usefulness in practice.

The purpose of this paper is to derive a simple model which can be used to predict the acoustic performance of a bulk-reacting dissipative silencer at low frequencies.

With a similar aim in mind, Wang [4] applied the decoupling method to the case of a dissipative silencer with perforated tube (the decoupling method having been derived by Sullivan and Crocker [5] for the case of an expansion chamber with perforated tube). This method considers that the acoustic pressure is uniform on either side of the perforated tube and that the difference between those two pressures is linked to the perforated tube impedance. However, in most practical applications, the perforated tube is introduced to avoid erosion of the porous material and is not supposed to have any significant acoustical effect. However, when the impedance of the perforated tube is small or vanishes, the decoupling method fails to give an accurate description of the muffler.

The key point of the model developed in this paper is that it takes into account the real radial variation of pressure (and then of radial velocity) in an approximate way. In the presentation given here, this effect is introduced via an equivalent admittance linking the difference in mean pressures between the air and the material to the radial velocity at the interface. Then, even if the perforated tube is absent the model can give an accurate description of acoustical performance.

In Section **2** the principle of the method is described. By averaging the Euler and continuity equation both for the air and the porous material, an eigenequation is obtained by assuming that the radial velocity at the interface only depends on the difference in mean pressure. The equivalent admittance is then given by assigning an appropriate shape for the radial velocity profile. The eigenequation displays two kind of solutions in the lined section: one correspons to the classical plane or quasi-plane wave, the other takes into account most of the effects of higher order modes. This approach is applied in Section **3** to a sudden lined expansion. The model is applied to the case with neither porous material nor perforated tube to provide an easy comparison with an exact solution. The modeling of the cylindrical dissipative silencer of finite length is given in Section **4**. Predictions using the model are then compared to experimental results.

2. LOW FREQUENCIES APPROXIMATION

2.1. AVERAGED EQUATIONS

In this section, the basic linear equations governing the propagation of axisymmetric fluctuations in a duct of radius r_b are given. This duct consists of an inner cylinder with radius r_a, referred to as region A, and of an outer coaxial cylinder with radius varying from r_a to r_b, referred to as region B (see Figure 1). Between those two regions, a rigid perforated screen induces a pressure jump proportional to the radial velocity.

The fluctuating variables used here are the pressure p, the axial velocity u and radial velocity v with a subscript a or b depending on the region. In region A, the fluid is characterized by the compressibility κ_a and the density ρ_a. In region B, a porous material with a rigid frame can be present and is described by an equivalent fluid model. The porous media is then characterized by an effective compressibility $\kappa_b(\omega)$ and an effective density $\rho_b(\omega)$ depending on the frequency ω. Those effective values are given in appendix A as functions of the characteristics of the material and of the saturating fluid.

Taking a time dependence $e^{j\omega t}$, the propagation equations can be found from the continuity and the Euler equations:

$$j\omega\kappa_i \, p_i = -\nabla.\mathbf{v}_i \tag{1a}$$

$$j\omega \, \rho_i \, \mathbf{v}_i = -\nabla p_i \tag{1b}$$

where $i = a$ or b.

If there is no screen the radial velocity and the pressure are continuous at $r = r_a$. With a perforated screen (the impedance of the screen is z_s), the radial velocity is still continuous but the pressure jumps from $p_b(r_a)$ to $p_a(r_a)$ with $p_b(r_a) - p_a(r_a) = z_s v_a(r_a) = z_s v_b(r_a)$.

By integrating equation (1a) and the projection of equation (1b) on the axial direction, the following averaged equations are obtained:

$$Y_a\,\bar{p}_a = -\frac{dU_a}{dx} - q, \qquad Z_a\,U_a = -\frac{d\bar{p}_a}{dx}, \tag{2a}$$

$$Y_b\,\bar{p}_b = -\frac{dU_b}{dx} + q, \qquad Z_b\,U_b = -\frac{d\bar{p}_b}{dx}. \tag{2b}$$

where $Y_i = j\omega\kappa_i S_i$ is the admittance per unit length, $Z_i = j\omega\rho_i/S_i$ is the impedance per unit length, $\bar{p}_i$ is the mean pressure over the section S_i ($S_a = \pi r_a^2$ and $S_b = \pi(r_b^2 - r_a^2)$), U_i is the acoustical flow rate over the section S_i (integral of axial velocity over the section) and $q = 2\pi r_a\,v_a(r_a)$ is the flow rate per unit length through the perforated screen.

The flow rate q is supposed to be linearly related to the difference of mean pressure between the two regions: $q = Y(\bar{p}_a - \bar{p}_b)$. Thus, by taking a dependence in the axial direction having the form $e^{-jk\,x}$, two equations for the mean pressures can be found:

$$\begin{bmatrix} k^2 + \Gamma_a^2 + Z_a Y & -Z_a Y \\ -Z_b Y & k^2 + \Gamma_b^2 + Z_b Y \end{bmatrix} \begin{pmatrix} \bar{p}_a \\ \bar{p}_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

where the propagation constants Γ_a and Γ_b in region A and B are given by $\Gamma_i^2 = Z_i Y_i$ $(i = a, b)$.

2.2. DETERMINATION OF THE WAVENUMBERS

The determinant of the system (3) must vanish in order to have non trivial solutions. This gives the eigenequation for the wavenumber k :

$$k^4 + 2\left(\Gamma_m^2 + Z_m Y\right)k^2 + \left(\Gamma_m^2\right)^2 - \frac{1}{4}\left(\Delta\Gamma^2\right)^2 + 2\left(Z_m\Gamma_m^2 - \frac{1}{4}\Delta Z \Delta\Gamma^2\right)Y = 0 \tag{4}$$

where $\Gamma_m^2 = (\Gamma_a^2 + \Gamma_b^2)/2$, $\Delta\Gamma^2 = \Gamma_a^2 - \Gamma_b^2$, $Z_m = (Z_a + Z_b)/2$ and $\Delta Z = Z_a - Z_b$.

The solutions are written

$$\begin{aligned} k_1^2 &= -\Gamma_m^2 - Z_m Y(1 - A) \\ k_2^2 &= -\Gamma_m^2 - Z_m Y(1 + A) \end{aligned}$$

where

$$A = \left(1 + \frac{\Delta\Gamma^2}{2Z_m^2 Y}\left(\frac{\Delta\Gamma^2}{2Y} + \Delta Z\right)\right)^{1/2}. \tag{5}$$

The values of k_1 and k_2 are chosen so that their imaginary parts are negative (obviously, $-k_1$ and $-k_2$ are also solutions of the eigenequation). By substituting k_1 for k in equation (3), it can be seen that the mean pressures are related for the first solution by $\bar{p}_{b1} = m\bar{p}_{a1}$ where

$$m = \frac{\Delta\Gamma^2 + \Delta Z Y + 2Z_m A Y}{2Z_a Y} \tag{6}$$

and for the second solution $mZ_a\bar{p}_{b2} = -Z_b\bar{p}_{a2}$. When the porous material in the region B is not too resistive (i.e. when $\Delta\Gamma^2 \ll \Gamma_m^2$), the absolute value of m is close to 1. In this case the first solution corresponds to a quasi plane mode while the second solution corresponds to the effects of higher order modes.

2.3. DETERMINATION OF THE EQUIVALENT ADMITTANCE

The remaining problem is the determination of the admittance

$$Y = \frac{2\pi r_a\, v_a(r_a)}{\bar{p}_a - \bar{p}_b} = \frac{2\pi r_a}{j\omega\rho_a\delta_a + z_s + j\omega\rho_b\delta_b}$$

where

$$\delta_a = \frac{\bar{p}_a - p_a(r_a)}{j\omega\rho_a v_a(r_a)} \qquad \delta_b = \frac{p_b(r_a) - \bar{p}_b}{j\omega\rho_b v_b(r_a)}.$$

To compute the coefficients δ_a and δ_b, the shape of the pressure and of the radial velocity have to be known in the regions A and B. For that purpose approximate profiles of velocity and pressure are needed in the two regions. In the problem studied in this paper (area expansion), a good approximation of the acoustic field just behind an expansion is needed. Thus the approximate profiles are choosen with this aim in mind.

In the region A, the radial velocity vanishes when $r = 0$ and is supposed to increase linearly. Thus, $v_a(r)$ is approximated by $v_a(r) = A\, r$. By integration of the radial projection of equation (1b), the pressure $p_a(r)$ is given by $p_a(r) = p_{0a} - j\omega\rho_a A r^2/2$. In this approximation, the coefficient δ_a can easily be computed and $\delta_a = r_a/4$.

In the region B, the radial velocity is chosen **1)** to fulfill the boundary condition at the outer wall: $v_b(r_b) = 0$; **2)** to fit the radial velocity just behind an expansion with a big area ratio. Thus the axial velocity is taken with the form $v_b(r) = B(1/r_b^2 - 1/r^2)$ [6] and, by integration, the pressure is $p_b(r) = p_{0b} - j\omega\rho_b B(r/r_b^2 + 1/r)$. With this approximate shape, the coefficient δ_b is equal to $\delta_b = r_a f(\alpha)$ with $\alpha = r_a/r_b$ and

$$f(\alpha) = \frac{(1-\alpha)(3+\alpha)}{3(1+\alpha)^2}.$$

It can be noted that for moderate area ratio ($\alpha \sim 0.5$) a linear radial velocity profile in the region B give a value δ_b very close to the value obtained with the choosen profile.

Then the admittance Y is written

$$Y = \frac{2\pi}{j\omega\rho_a}\left[\frac{1}{4} + \frac{\rho_b}{\rho_a} f(\alpha) + \frac{z_s}{j\omega\rho_a r_a}\right]^{-1}. \tag{7}$$

2.4. MODEL WITHOUT POROUS MATERIAL

In this subsection, the region B is filled with the same fluid as the region A. The sound velocity in the fluid is c_0 and ρ_0 is the density. Then the lineic admittances are $Z_a = j\omega\rho_0/S_a$ and $Z_b = j\omega\rho_0/S_b$. The propagation constants are $\Gamma_a = \Gamma_b = \Gamma_m = -j\omega/c_0$ and $\Delta\Gamma^2 = 0$. The wavenumbers are

$$k_1 = j\Gamma_m = \frac{\omega}{c_0} \tag{8}$$

and

$$k_2 = (-\Gamma_m^2 - 2Z_m Y)^{1/2} = \left(\left(\frac{\omega}{c_0} \right)^2 - j\omega\rho_0 \left(\frac{1}{S_a} + \frac{1}{S_b} \right) Y \right)^{1/2}. \tag{9}$$

It can easily be seen that $\bar{p}_{a1} = \bar{p}_{b1}$. Thus, the first solution corresponds to the classical plane wave in the duct and is not influenced by the value of the admittance Y. For the second solution, the mean pressures in both regions are related by $S_a\bar{p}_{a2} = -S_b\bar{p}_{b2}$.

Equation (9) is given by Pierce [7] when there is a perforated tube between the two regions and can be deduced from the results of Kergomard *et al.* [8] in the case of a perforated tube modelled discretely. The main difference is that the model presented here takes into account in an approximate form the difference between the actual pressure and the mean pressure in the two regions. This difference is included in the admittance Y. Thus, this model is also valid when the screen impedance goes to zero.

When the perforated screen is absent $z_s = 0$, the admittance Y is given by

$$Y = \left(\frac{j\omega\rho_0}{2\pi} \left(\frac{1}{4} + f(\alpha) \right) \right)^{-1}$$

and the wave number of the second solution is

$$k_2 = \left(\left(\frac{\omega}{c_0} \right)^2 - \frac{\gamma^2}{r_b^2} \right)^{1/2} \qquad \text{where} \qquad \gamma^2 = \frac{2}{\alpha^2(1-\alpha^2)(1/4 + f(\alpha))}.$$

This second mode is evanescent as long as the pulsation frequency ω is lower than $\omega_c = \gamma c_0/r_b$. For $\alpha = 0.5$, the value of ω_c is $4.43\, c_0/r_b$ which is reasonably close to the cut-off pulsation frequency $3.83\, c_0/r_b$ for the second axisymmetric mode of a circular duct with radius r_b. This comes from the similarity of the pressure profile of the second approximate solution and of the pressure profile of the exact second mode with $\alpha = 0.5$ (see Figure 2a). When α is much smaller than 1 (see Figure 2b), those profiles are very different. It can be seen from this figure that the second approximate solution is especially adapted to take into account most of the effects of the higher order modes near a sudden expansion.

3. SUDDEN EXPANSION AT LOW FREQUENCIES

3.1. TRANSFERT MATRIX OF AN EXPANSION

Two semi-infinite ducts of radius r_a and r_b are joined at $x = 0$. In the small duct ($x < 0$, radius r_a) only a plane wave propagates: $p_0 = p_0^+ e^{-jk_0x} + p_0^- e^{jk_0x}$ where $k_0 = \omega/c_0$. This wave induces an acoustical flow rate equal to U_0 at $x = 0$ with $Z_a U_0 = jk_0(p_0^+ - p_0^+)$. In the lined duct ($x > 0$, radius $r_b > r_a$), the model proposed in section **2.** is applied when the frequency is lower than ω_c. In the lined duct, the termination for the evanescent mode (subscript 2) is supposed to be anechoic. Thus the wave in this duct can be seen as the sum of three terms with an axial dependence e^{-jk_1x}, e^{jk_1x} and e^{-jk_2x} (the imaginary part of $k_{1,2}$ having been taken negative). Accordingly, the mean pressures in the regions A and B of the lined duct are written:

$$\begin{aligned} \bar{p}_a &= p_1^+ e^{-jk_1x} + p_1^- e^{jk_1x} + p_2^+ e^{-jk_2x} \\ \bar{p}_b &= m p_1^+ e^{-jk_1x} + m p_1^- e^{jk_1x} - \frac{Z_b}{mZ_a} p_2^+ e^{-jk_2x} \end{aligned}$$

where m is given by equation (6).

The boundary conditions at $x = 0$ are the continuity of the mean pressure and flow rate for $r < r_a$ and vanishing of the flow rate for $r_a < r < r_b$:

$$p_0^+ + p_0^- = p_1^+ + p_1^- + p_2^+; \tag{10}$$

$$jk_0 \, (p_0^+ - p_0^-) = jk_1 \, (p_1^+ - p_1^-) + jk_2 \, p_2^+; \tag{11}$$

$$0 = jk_1 m \, (p_1^+ - p_1^-) - jk_2 \frac{Z_b}{mZ_a} \, p_2^+. \tag{12}$$

From Eqs. (11) and (12), a continuity of volume velocity can be written $U_0 = U_1$ where $U_0 = (p_0^+ - p_0^-)/z_{c0}$, $z_{c0} = Z_a/jk_0 = \rho_0 c_0/S_a$ and $U_1 = (p_1^+ - p_1^-)/z_{c1}$ where $z_{c1} = Z_a Z_b / jk_1(Z_b + m^2 Z_a)$.

It can be noticed that U_0 is the real volume velocity in the small tube but that U_1 is not the total volume velocity of the first mode in the large tube. This real volume velocity associated with the quasi plane mode in the lined duct by is $\tilde{U}_1 = U_{a1} + U_{b1} = jk_1(1 + mZ_a/Z_b)(p_1^+ - p_1^-)/Z_a$. The relation between U_0 and $\tilde{U}_1$ is

$$\tilde{U}_1 = \frac{1 + mZ_a/Z_b}{1 + m^2 Z_a/Z_b} U_0. \tag{13}$$

Thus, the two total volume velocities coincide only when $m = 1$, i.e. when the porous material is absent, or when $m = 0$, i.e. when the porous material is so resistive that there is no acoustical flow through it.

Similarly, the relation between the mean pressure in the small tube $p_0 = p_0^+ + p_0^-$ and the mean pressure associated with the quasi-plane mode in the region A of the lined duct $p_1 = p_1^+ + p_1^-$ is

$$p_1 = p_0 - z_{add} U_0 \tag{14}$$

where

$$z_{add} = \frac{m^2 Z_a/Z_b}{1 + m^2 Z_a/Z_b} \frac{Z_a}{jk_2}.$$

This leads to the transfer matrix for the plane waves

$$\begin{pmatrix} p_1 \\ U_1 \end{pmatrix} = \begin{bmatrix} 1 & -z_{add} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} p_0 \\ U_0 \end{pmatrix}. \tag{15}$$

3.2. EXPANSION WITHOUT POROUS MATERIAL

When the region B is filled with the same fluid as the region A, equation (13) shows that the continuity of volume velocities between the plane modes on both side of the expansion ($U_0 = U_1$) can be applied in this approximate model. It can be noted that this relation is also verified in the exact model (see for instance [9]).

From equation (14), the plane mode pressures in the two ducts can be related by $p_1 = p_0 - j\omega\rho_0\Delta L(\omega)u_0$ where u_0 is the acoustic velocity of the plane mode in the small tube $u_0 = U_0/S_a$ and $\Delta L(\omega)$ is the added length given by

$$\Delta L = \frac{(1-\alpha^2)}{jk_2}$$

where jk_2 is a real and positive number if a resistive screen is absent (real part of $z_s = 0$). Without any screen and when $\omega \to 0$, this added length tends toward

$$\Delta L = r_a \frac{1-\alpha^2}{\alpha\gamma} = r_a \left(\frac{(1-\alpha)^2(1-\alpha^2)(15-2\alpha-\alpha^2)}{24} \right)^{1/2} \quad (16)$$

This result is compared in Figure 3 with the formulae for the added length given in reference [9] with a precision of 0.1 % for the zero-frequency limit. The agreement is good over all the α range.

Thus, this very simple model allows a good approximation of the acoustic behavior of a sudden expansion at low frequencies.

4. DISSIPATIVE SILENCER

4.1. TRANSFERT MATRIX OF AN EXPANSION CHAMBER

An expansion chamber of length L is filled with an annular porous material $r_b > r > r_a$. This chamber is connected with two pipes of radius r_a (see Figure 4). The approximate model developed above is applied in this chamber. The wave in the lined chamber can be seen as the sum of four terms with an axial dependence e^{-jk_1x}, e^{jk_1x}, e^{-jk_2x} and e^{-jk_2x}. The boundary conditions are the vanishing of the mean axial velocity in the porous media at $x = 0$ and $x = L$ and the continuity of the mean velocity and pressure in zone A at $x = 0$ and $x = L$. Thus, the problem can be completely solved in the context of the present approximate model. Nevertheless, significant simplifications appear when the amplitude of the most attenuated mode (wavenumber k_2) created at one side of the chamber can be neglected when it reaches the other side (i.e. $|e^{-jk_2L}| \ll 1$). This assumption is true for most practical applications. In this case, the transfer matrix of an expansion chamber can be seen as the product of the transfer matrices of an expansion, $\mathcal{T}_e$, of a propagation of the quasi plane wave from 0 to L, $\mathcal{T}_p$, and of a contraction, $\mathcal{T}_c$, where

$$\begin{pmatrix} p_I \\ U_I \end{pmatrix} = \mathcal{T}_e \begin{pmatrix} p_0 \\ U_0 \end{pmatrix} \quad \text{with} \quad \mathcal{T}_e = \begin{bmatrix} 1 & -z_{add} \\ 0 & 1 \end{bmatrix},$$

$$\begin{pmatrix} p_{II} \\ U_{II} \end{pmatrix} = \mathcal{T}_p \begin{pmatrix} p_I \\ U_I \end{pmatrix} \quad \text{with} \quad \mathcal{T}_p = \begin{bmatrix} \cosh(jk_1L) & -z_{c1}\sinh(jk_1L) \\ -\sinh(jk_1L)/z_{c1} & \cosh(jk_1L) \end{bmatrix}$$

and

$$\begin{pmatrix} p_t \\ U_t \end{pmatrix} = \mathcal{T}_c \begin{pmatrix} p_{II} \\ U_{II} \end{pmatrix} \quad \text{with} \quad \mathcal{T}_c = \begin{bmatrix} 1 & -z_{add} \\ 0 & 1 \end{bmatrix}.$$

The transfer matrix of the chamber can be written:

$$\begin{pmatrix} p_t \\ U_t \end{pmatrix} = \mathcal{T}_c\mathcal{T}_p\mathcal{T}_e \begin{pmatrix} p_0 \\ U_0 \end{pmatrix} = \begin{bmatrix} A & (A^2-1)/C \\ C & A \end{bmatrix} \begin{pmatrix} p_0 \\ U_0 \end{pmatrix} \tag{17}$$

where $A = \cosh(jk_1l) + z_{add}\sinh(jk_1l)/z_{c1}$ and $C = -\sinh(jk_1l)/z_{c1}$. The transmission and reflection coefficients of this chamber are

$$T = \frac{-2Z_{c0}C}{(Z_{c0}C - A)^2 - 1} \quad \text{and} \quad R = \frac{A^2 - (Z_{c0}C)^2 - 1}{(Z_{c0}C - A)^2 - 1}.$$

4.2. EXPERIMENTAL VALIDATION

The transmission and reflection coefficients of an expansion chamber with porous material were investigated experimentally in order to test the validity of the approximate model.

The chamber was inserted in a tube in which linear acoustical plane waves were propagated. On one side (side 0, see Figure 4), an acoustic source provided a wave in the frequency range 50–1500 Hz. The acoustic pressure in tube 1 is written $p_0 = p_0^+ e^{-j\tilde{k}_0 x} + p_0^- e^{j\tilde{k}_0 x}$ where p_0^+ and p_0^- are the incident and reflected pressures on side

0, $\tilde{k}_0$ is the wave number in the tube taking into account the viscothermal attenuation and x is the axial distance from side 0 of the chamber. Four microphones in tube 0 allow an overdetermined estimation of p_0^+ and p_0^-. The overdetermination is used, with a least square method, to increase the accuracy of the experimental results. On the other side (side t), four other microphones are used so that the transmitted pressure p_t^+ and the pressure reflected from the tube termination p_t^- can be determined on side t of the chamber.

Reciprocity and symmetry of the measured element imply that the transmission T and reflection R coefficients satisfy to $p_t^+ = T p_0^+ + R p_t^-$ and $p_0^- = T p_t^- + R p_0^+$. Thus, the coefficients

$$R = \frac{p_0^+ p_0^- - p_t^+ p_t^-}{{p_0^+}^2 - {p_t^-}^2} \quad \text{and} \quad T = \frac{p_0^+ p_t^+ - p_0^- p_t^-}{{p_0^+}^2 - {p_t^-}^2}$$

can be computed from the microphone data as functions of the frequency.

For the given expansion chamber the inner radius is $r_a = 15$ mm, the outer radius is $r_b = 47$ mm and the length is $L = 360$ mm. The porous material is a mineral wool whose acoustical parameters have been measured using other setup. The values of the parameters (see Appendix A) are: porosity $\Phi = 0.99$, tortuosity $\alpha_\infty = 1.1$, resistivity $\sigma = 75000$ kg m^{-3} s^{-1}, viscous and thermal characteristic lengths $\Lambda = 1\ 10^{-4}$ m and $\Lambda' = 2\ 10^{-4}$ m.

The mineral wool is known to be anisotropic. The resistivity perpendicular to the fibers (radial direction) is bigger than the resistivity along the fibers (axial direction). The ratio between the axial and radial resistivity is chosen to be 0.7 in accordance with [10]. The anisotropy can be easily introduced in the approximate model. The effective density to be used depends on the direction on which the Euler equation is projected. The Euler equation in the radial direction only appears in the determination of the equivalent admittance Y. Thus, the effective density in Equation (7) is computed with the radial parameters and the other densities are computed with the axial parameters.

The results from measurement (circles) and prdicted using the approximate model (continuous lines) are shown in Figure 5. For comparison the results for the empty chamber are given as the dashed lines. The agreement is very good except on the absolute value of the transmission when the frequency is above 1 kHz. It should be noted that the attenuation is of the order of 60 dB in this region. Some experimental problems, like flanking transmission through the external tube of the chamber, could explain this discrepancy. Nevertheless, it can be concluded that the approximate model gives an accurate description of the performance of the dissipative silencer.

5. CONCLUSIONS

A method based on the approximation of the radial pressure profile is developed to analyse the acoustic performance of an expansion and of an expansion chamber at low frequencies. This model is able to predict very accurately the added length of an expansion when there is neither porous material nor perforated tube and can be applied as well when a bulk-reacting lining and perforated tube are included. This model of a lined expansion chamber gives results which compare very favorably with experimental data. This approach could be extended to the case where a flow is present [11] and it can describe the appearance of hydrodynamic modes in this case.

Owing to its simplicity, this approximate model of a dissipative silencer could be easily implemented as a predictive tool for computing the acoustic performances in flow duct acoustics.

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APPENDIX

In rigid-framed porous materials, the linear sound propagation can be described by means of using an equivalent fluid with an effective density and an effective compressibility which are complex values depending on the frequency (see, for example, [12] and [13]). The continuity and Euler equations are written

$$\kappa_e \frac{\partial p}{\partial t} = -\nabla.\mathbf{v}$$
$$\rho_e \frac{\partial \mathbf{v}}{\partial t} = -\nabla p$$

where p and $\mathbf{v}$ are the macroscopic acoustic pressure and velocity (the macroscopic velocity is chosen such as the continuity of velocity applies at an interface between the porous material and air).

The effective characteristics of the material κ_e and ρ_e can be obtained with the help of six parameters: the porosity Φ, the tortuosity α_∞, the viscous and thermal permeabilities k_0 and k_0', the viscous and thermal characteristic lengths Λ and Λ'. The effective density is equal to

$$\rho_e = \frac{\rho_0 \alpha_\infty}{\Phi} \left(1 + \frac{1}{jx} \left[1 + \frac{M}{2} jx \right]^{1/2} \right)$$

where the reduced frequency x is given by

$$x = \frac{\omega \alpha_\infty k_0}{\nu \Phi},$$

the shape factor is

$$M = \frac{8 k_0 \alpha_\infty}{\Phi \Lambda^2},$$

ρ_0 is the density and ν is the kinematic viscosity of the fluid. The effective compressibility is equal to

$$\kappa_e = \frac{\Phi}{\rho_0 c_0^2} \left(\gamma - (\gamma - 1) \left(1 + \frac{1}{jx'} \left[1 + \frac{M'}{2} jx' \right]^{1/2} \right)^{-1} \right)$$

where the reduced frequency x' is

$$x' = \frac{\omega P_r k'_0}{\nu \Phi},$$

the shape factor is

$$M' = \frac{8k'_0}{\Phi \Lambda'^2},$$

c_0 is the sound velocity and P_r is the Prantl number in the fluid. In this paper, the static thermal permeability is approximated [13] by $k'_0 = \Phi \Lambda'^2/8$ and the static viscous permeability k_0 is related to the air flow resistivity by $\sigma = \rho_0 \nu / k_0$.

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FIGURES CAPTION

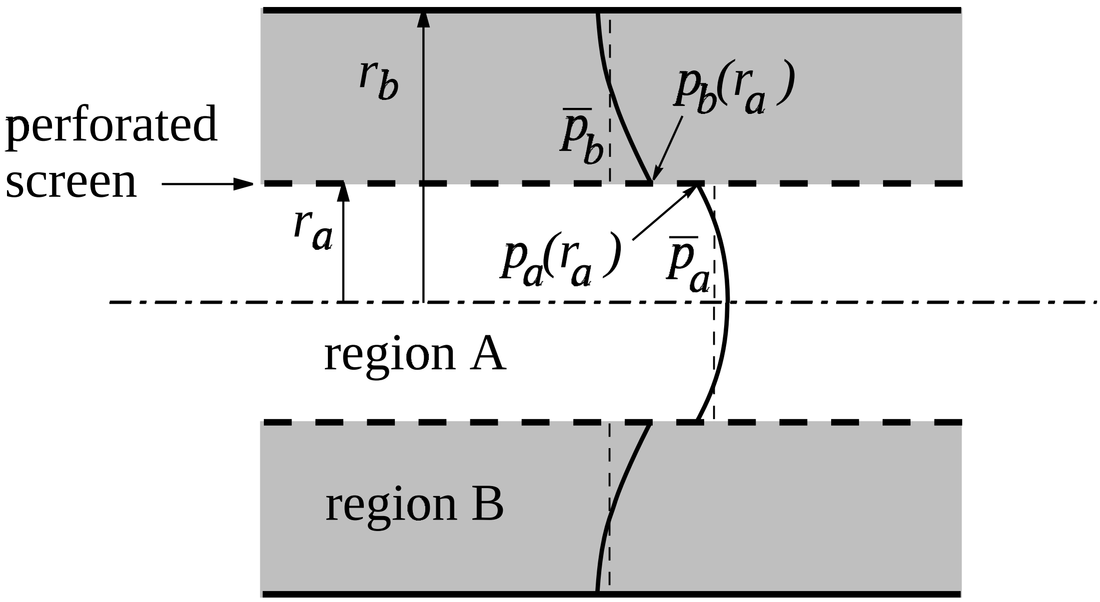
Figure 1: Typical geometry and pressure profile.

Figure 2: Normalized pressure for the second solution $p_2/\bar{p}_a$ as a function of the normalized radius r/r_b with $\alpha = 0.5$ **(a)** and $\alpha = 0.05$ **(b)**; continuous line: approximate solution, dotted line: mean pressure in regions A and B; dashed line: second axisymmetric mode.

Figure 3: Variation of the added length $\Delta L/r_a$ at the zero frequency limit as a function of the radius ratio $\alpha = r_a/r_b$; continuous line: equation (16), dashed line: reference [9].

Figure 4: Geometry of the lined expansion chamber.

Figure 5: Absolute value of the reflection (a) and transmission (b) coefficients of an expansion chamber with porous material. $\circ$: measurements, continuous line : approximate model, dashed line : results for the same chamber without porous material.



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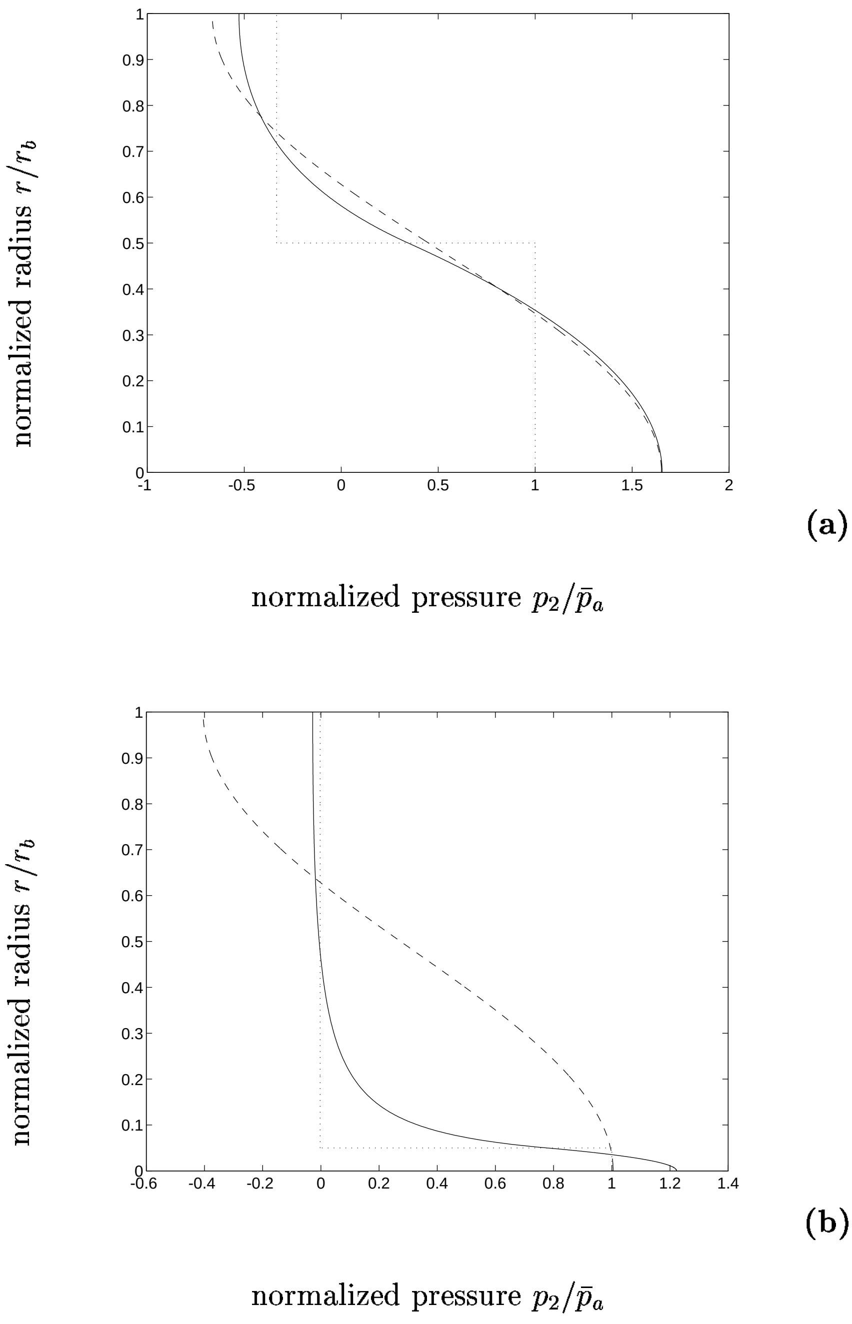
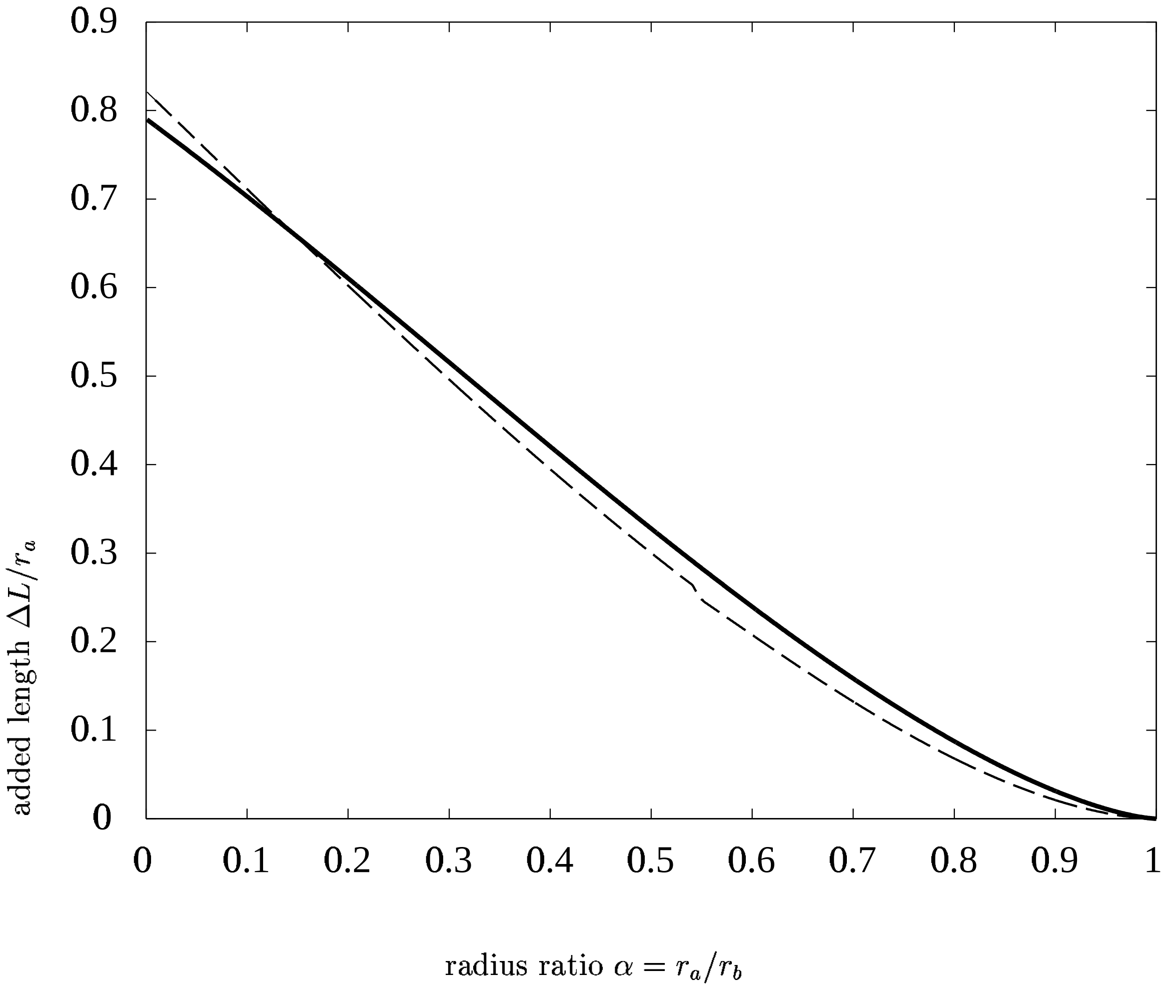


Figure 2: Normalized pressure for the second solution $p_2/\bar{p}_a$ as a function of the normalized radius r/r_b with $\alpha = 0.5$ **(a)** and $\alpha = 0.05$ **(b)**; continuous line: approximate solution, dotted line: mean pressure in regions A and B; dashed line: second axisymmetric mode.



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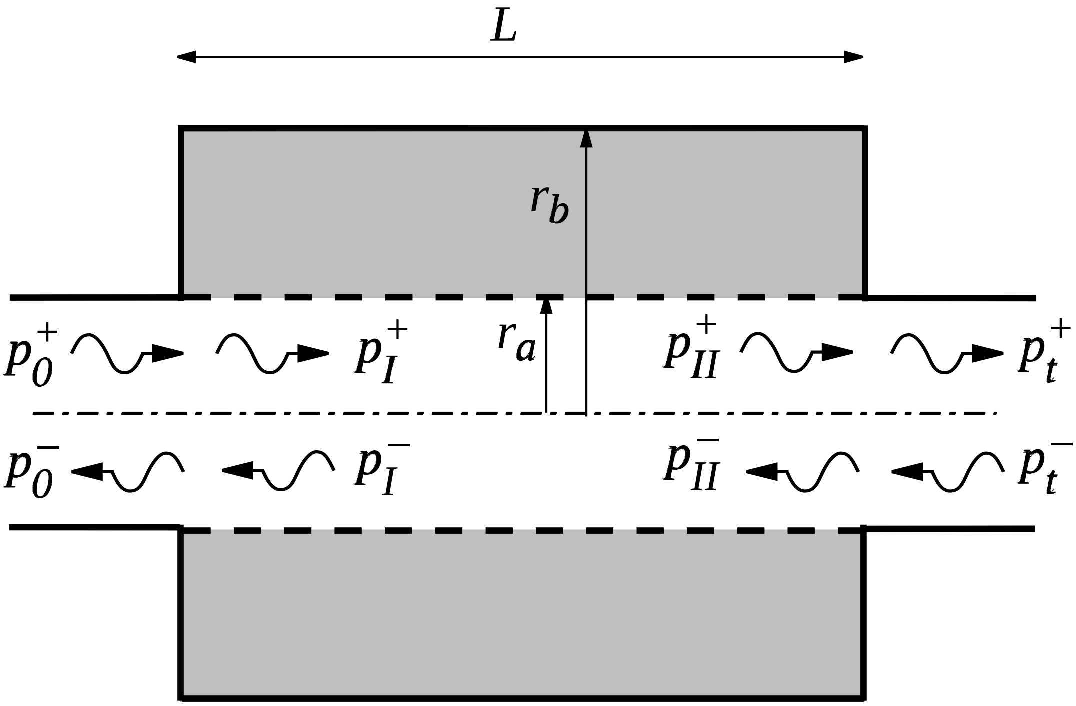

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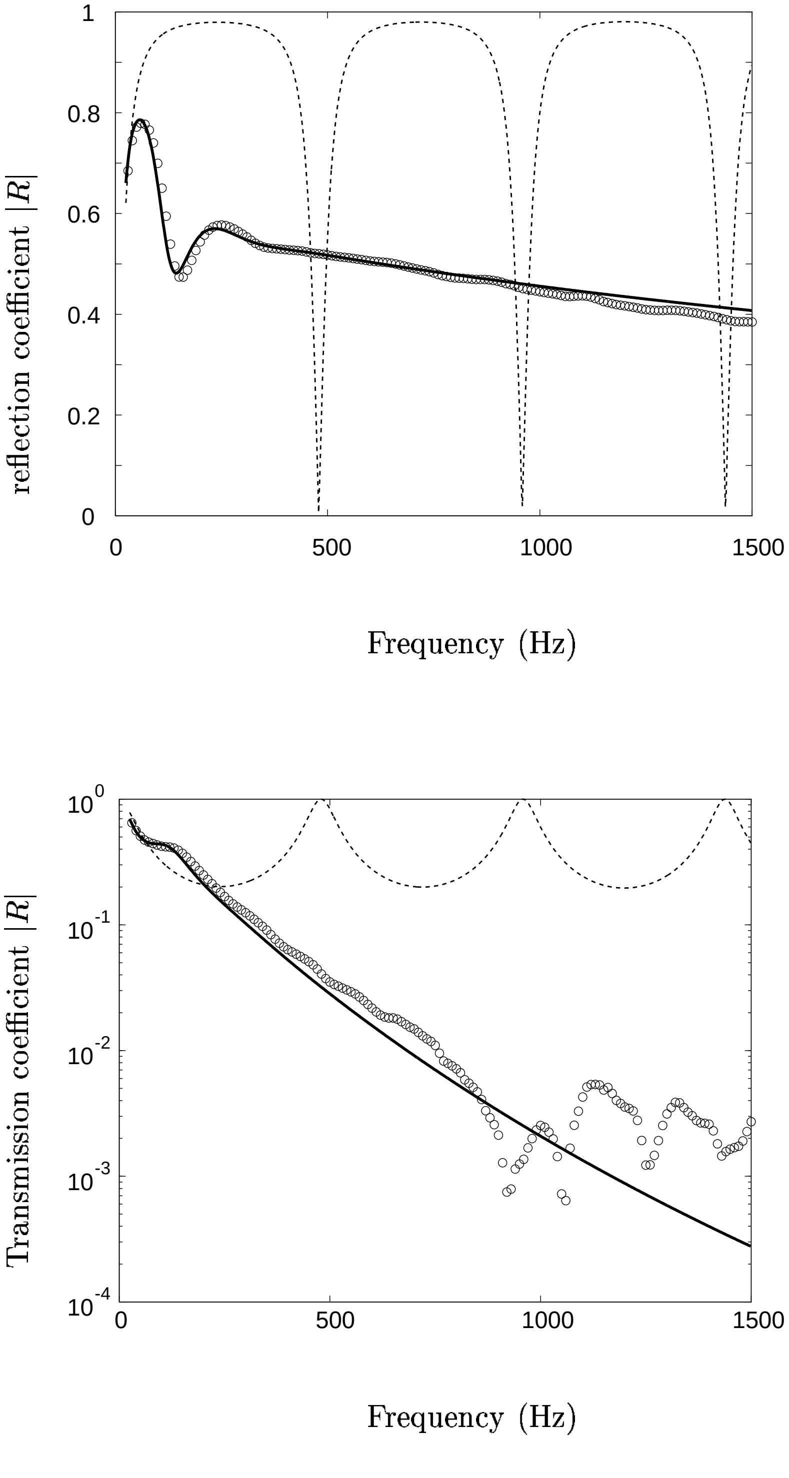

(a)

(b)

Figure 5: Absolute value of the reflection (a) and transmission (b) coefficients of an expansion chamber with porous material. ∘ : measurements, continuous line : approximate model, dashed line : results for the same chamber without porous material.