Determination of the acoustical energy dissipation/production potentiality from the acoustical transfer functions of a multiport

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Abstract

The acoustic energy conservation law in presence of flow can be very effectively expressed with the help of two variables: the acoustic mass velocity and the acoustic exergy. The minimal and maximal ratio of the dissipated or generated acoustic power over the incident acoustic power are computed from the scattering matrix of an acoustic element. This method is applied to a diaphragm and to a laminar element in presence of flow. It is shown that the results provide indication for the physics of dissipation of acoustic energy in these sytems.

1 Introduction

One considers any acoustic element located at the junction of waveguides (see Figure 1). In each guide, a uniform mean flow is assumed and several acoustic modes can propagate. The relations between the incoming waves and the outcoming waves (both plane and transverse) are given by a scattering matrix. The aim of this paper is to determine from a given acoustical scattering matrix the potentiality of the element to dissipate or generate acoustic energy i.e. to determine the minimal and maximal ratio of the dissipated or generated acoustic power over the incident acoustic power.

In a first section, conservation law for acoustic energy are derived in the natural variables: acoustic mass velocity and acoustic exergy. The second section shows how to extract, from the scattering matrix, the minimal and maximal ratio of the dissipated acoustic energy over the incident acoustic energy. Two examples are then given in which this ratio is computed for the measured scattering matrices of a diaphragm and of a laminar element in presence of flow.

2 Acoustic power in waveguides in presence of flow

The conservation law for acoustic energy in presence of flow was given by Morfey [1] under general assumptions. It can be put in the form

$$\frac{\partial E}{\partial t} + \nabla.\mathbf{I} = -\mathcal{D} \tag{1}$$

where the density E and flux $\mathbf{I}$ of acoustic energy function are given respectively by

$$E = \frac{1}{2}\left(\mathbf{v}'.\mathbf{m}'_a + \rho'_a\,\Pi'\right)$$

and

$$\mathbf{I} = \Pi'\,\mathbf{m}'_a$$

while $\mathcal{D}$ represents the rate of acoustic energy dissipation per unit of volume.

In those expressions $\mathbf{v}'$ is the irrotational fluctuating velocity, ρ'_a is the acoustic part of the fluctuating density ($\rho'_a = p'/c_0^2$). The acoustic mass velocity $\mathbf{m}'_a$ and the fluctuating part of the total external mechanical energy (acoustic exergy) Π' are given by

$$\mathbf{m}'_a = \rho_0\,\mathbf{v}' + \rho'_a\,\mathbf{v}_0$$

and

$$\Pi' = p'/\rho_0 + \mathbf{v}_0.\mathbf{v}'.$$

The two thermodynamic variables $\mathbf{m}'_a$ (extensive) and Π' (intensive) are very appropriate to study acoustic propagation in presence of flow. The product of these variables gives the flux of acoustic energy (like the classical variables $\mathbf{u}'$ and p' without flow), then all the formalism of the transmission lines can be applied to these variables in presence of flow [2], [3].

It could be noted that E is not a real density of energy because this quadratic form is not positive definite and in some cases of supersonic flow can be negative [2], [4]. Nevertheless, compared to the true energy, this form presents the advantage that $\mathcal{D} = 0$ when the process is isentropic.

The equation (1) will be applied in its integrated and time averaged form to the junction of i pipes

$$\sum_i W^i = \int_V < \mathcal{D} > \ dV \tag{2}$$

where

$$W^i = \int_{S_i} < m'_x \, \Pi' > dS \tag{3}$$

is the sound power flowing out of the volume V through the cross-section S_i of the waveguide number i, see figure 1, (m'_x is the projection of $\mathbf{m}'_a$ on the normal to the surface) and the sum is performed over all the waveguides.

In most practical cases, one can divide any acoustic element in presence of flow in two different regions: (i) an internal region of strong interaction between hydrodynamic flow and acoustic: the volume V in equation (2), (ii) an outer region far upstream and downstream of the perturbing element where both hydrodynamic and acoustic flow are again "fully developed". In this second region, i. e. in the waveguides, the main flow is assumed to be axial and uniform across the section (the Mach number is given by $M = u_0/c_0$ where u_0 is the axial main velocity).

With those asumptions the entropy and vorticity perturbations (pseudo-sound) in the waves guides are only convected by the main flow and can be treated separately from the acoustic pressure waves (sound) by introducing entropy and vorticity currents [2], [3]. The acoustical part of the sound transmission is described by classical wave equation for acoustical exergy Π' [5]

$$\left(\frac{1}{c_0^2} \frac{D_0^2}{Dt^2} - \nabla^2 \right) \Pi' = 0. \tag{4}$$

A set of transverse orthonormal eigenfunctions $\psi_m(\mathbf{y})$, associated with axial wave numbers $k_m^\pm$ can be used to expand Π' in a series:

$$\Pi' = e^{i\omega t} \sum_m \Pi_m(x)\psi_m(\mathbf{y}) \tag{5}$$

in which following Morfey [6] we have:

$$\begin{aligned} \Pi_m(x) &= \Pi_m^+ + \Pi_m^- \\ &= \frac{1}{S_i} \int_{S_i} \Pi'(x, \mathbf{y})\psi_m(\mathbf{y})dy. \end{aligned}$$

and

$$k_m^\pm = \frac{\omega}{c_0} \left(\frac{\pm\alpha_m - M}{1 - M^2} \right)$$

where

$$\alpha_m = \left(1 - \left(\gamma_m \frac{c_0}{\omega} \right)^2 (1 - M^2) \right)^{1/2}.$$

The γ_m are the discrete eigenvalues of these eigenfunctions.

In the same way, we can define the fluctuating mass flow rate associated with mode m at section S_i by

$$M_m(x) = M_m^+ - M_m^- = \int_{S_i} m'_x(x, \mathbf{y})\psi_m(\mathbf{y})dy.$$

The forward and backward mass flow rates $M_m^\pm$ are given, for the mode m, by

$$M_m^\pm = Y_m \Pi_m^\pm$$

where $Y_m = \alpha_m \rho_0 S_i / c_0$.

The forward and backward sound power, for the mode m, are calculated from equation (3)

$$W_m^\pm = 1/2 \ \mathrm{Re}\left(\int_{S_i} m_m^\pm \overline{\Pi_m^\pm} dS\right)$$

where the bar indicates complex conjugate and Re( ) is the real part. With the above relations, these sound powers are given by

$$W_m^{\pm} = \pm Y_m \frac{|\Pi_m^{\pm}|^2}{2} = \pm Y_m \tilde{\Pi}_m^{\pm\,2}$$

(the tilde indicates an effective value) above the cut-off frequency of mode m (i. e. when α_m is real) and

$$W_m^{\pm} = 0$$

below the cut-off frequency (i. e. when α_m is imaginary).

The energy dissipated in the volume V is the difference on all the propagating modes between the energy flow of incident and scattered waves. For each propagating waves, incident and scattered amplitudes a_m and b_m are defined such that $W_m^{inc} = \overline{a}_m a_m$ and $W_m^{sca} = \overline{b}_m b_m$ with

$$a_m = {Y_m}^{1/2} \tilde{\Pi}_m^{+}, \quad b_m = {Y_m}^{1/2} \tilde{\Pi}_m^{-}.$$

Thus, denoting $\mathbf{a}$ and $\mathbf{b}$ the column vectors of component a_m and b_m, the acoustic power dissipated in the volume V can be calculated by

$$P_{dis} = {}^t\overline{\mathbf{a}}\mathbf{a} - {}^t\overline{\mathbf{b}}\mathbf{b} = {}^t\overline{\mathbf{a}}\mathbf{a} - {}^t\overline{\mathbf{a}}({}^t\overline{\mathsf{S}}\mathsf{S})\mathbf{a} \tag{6}$$

where S is the scattering matrix such as $\mathbf{b} = \mathsf{S}\mathbf{a}$ and ${}^t\overline{\mathsf{S}}$ is the adjoin matrix of S (Hermitian transpose).

3 Potentiality of acoustic energy dissipation or generation

If S is an unitary matrix (${}^t\overline{\mathsf{S}} = \mathsf{S}^{-1}$), acoustic power dissipation or generation is absent ($P_{dis} = 0$). In the general case, dissipation or generation depend on the non-unitary character of S and on the phase and amplitude relations between the incident waves.

Without loss of generality, the sum of the energy of the incident waves can be normalized to 1. In this case, the reflected part of the energy is given by the value of the quadratic form ${}^t\overline{\mathbf{a}}({}^t\overline{\mathsf{S}}\mathsf{S})\mathbf{a}$ on the multidimensional sphere of radius 1 (${}^t\overline{\mathbf{a}}\mathbf{a} = 1$). This quadratic form can be reduced to a sum of squares [7]:

$$ {}^t\overline{\mathbf{a}}({}^t\overline{\mathsf{S}}\mathsf{S})\mathbf{a} = \sum_i \lambda_i |d_i|^2, \tag{7} $$

where the λ_i are the eigenvalues of the positive definite Hermitian matrix ${}^t\overline{\mathsf{S}}\mathsf{S}$, and thus real and positive. The vector $\mathbf{d}$ is given by $\mathbf{d} = {}^t\overline{\mathsf{T}}\mathbf{a}$ where T is the unitary matrix which transforms ${}^t\overline{\mathsf{S}}\mathsf{S}$ into the diagonal matrix of the eigenvalues: ${}^t\overline{\mathsf{T}}({}^t\overline{\mathsf{S}}\mathsf{S})\mathsf{T} = \mathrm{diag}(\lambda_1,\ \lambda_2,\ \ldots)$.

The sphere of radius 1 in variables $\mathbf{a}$ is transform by T in an other sphere of radius 1 in the new variables $\mathbf{d}$ (${}^t\overline{\mathbf{d}}\mathbf{d} = 1$). In these new variables, the dissipated power is given by

$$ P_{dis} = \sum_i (1 - \lambda_i)|d_i|^2 = \sum_i \xi_i |d_i|^2. \tag{8} $$

The maximum and minimum value of λ_i show respectively the minimum and maximum of the potentially reflected energy and the values

$$ \xi_{\min} = 1 - \lambda_{\max} $$

and

$$\xi_{\max} = 1 - \lambda_{\min}$$

show the minimum and maximum of the potentially dissipated energy [2].

A system is passive if the acoustic energy of the incoming waves is always greater than the acoustic energy of the outcoming waves and so if the quadratic form (8) is always positive. This can be obtain if and only if all the eigeinvalues of ${}^t\overline{\mathsf{S}}\mathsf{S}$ are smaller than 1.

The eigenvectors of ${}^t\overline{\mathsf{S}}\mathsf{S}$, related to $\xi_{\max}$ (respectively $\xi_{\min}$), give the relations to impose to the incident waves to have a maximal (respectively minimal) part of the incident acoustic power dissipated.

4 Illustrative examples

In this section, we use the above description to investigate the dissipated power in a laminar element and in a diaphragm. The scattering matrices of those elements have been measured with a superimposed mean flow at low frequencies when only plane waves can propagate in the upstream and downstream waves guides which have the same section.

A 2×2 scattering matrix relates the upstream exergy $\Pi_1^{\pm}$ to the downstream exergy $\Pi_2^{\pm}$ by

$$\begin{pmatrix} \Pi_1^- \\ \\ \Pi_2^- \end{pmatrix} = [\mathsf{S}] \begin{pmatrix} \Pi_1^+ \\ \\ \Pi_2^+ \end{pmatrix} = \begin{bmatrix} R^+ & T^- \\ \\ T^+ & R^- \end{bmatrix} \begin{pmatrix} \Pi_1^+ \\ \\ \Pi_2^+ \end{pmatrix}. \tag{9}$$

In this case, the exergy is simply related to the pressure of travelling waves by

$$\Pi_i^{\pm} = (1 \pm M_i) p_i^{\pm} / \rho_0.$$

For plane wave propagation in an uniform incompressible mean flow, the use of exergy is equivalent to the use of convected pressure [8] ($p'_c = p' + \rho_0 u_0 u'$) but differs from the use of the stagnation enthalpy [9] ($B' = p'/\rho_0 + u_0 u' + T_0 s'$) because the term involving entropy fluctuations $T_0 s'$ which is generated by irreversible process cannot always be neglected.

The elements of the S matrix could be seen as anechoic pressure transmission coefficient:

$$T^+ = p_2^+/p_1^+ \quad \text{when} \quad p_2^- = 0,$$

$$T^- = p_1^-/p_2^- \quad \text{when} \quad p_1^+ = 0$$

and modified anechoic pressure reflection coefficient:

$$R^+ = \frac{1-M}{1+M}\frac{p_1^-}{p_1^+} \quad \text{when} \quad p_2^- = 0,$$

$$R^- = \frac{1+M}{1-M}\frac{p_2^+}{p_2^-} \quad \text{when} \quad p_1^+ = 0.$$

The schematic description of the experimental set-up is displayed on Figure 2 (for a detailed description of the set-up see [10]). The frequency range experimentally investigated is 20–800 Hz to ensure that only plane waves can propagate in the upstream and downstream waveguide of diameter 30 mm. The elements of the S matrix were determined by a two sources method [11]. The scattering matrix was measured between the first microphone upstream and the first microphone downstream, then the effect of the propagation of the planar mode between the element and the microphones (L_1 and L_2 on Figure 2) were subtracted. L_1 and L_2 are respectively about 12 and 24 times the diameter of the tube. Those large values are meant to avoid measuring in the internal region where the pseudo-sound is significant.

4.1 diaphragm in presence of flow

Two circular diaphragms of diameter 15 mm inserted in a tube of diameter 30 mm were investigated. One of this diaphragm has rounded edges (radius of curvature 1 mm for a thickness of 2 mm) and the other has sharp edges. The values of ξ_{min} and ξ_{max} which represent the minimal and maximal possible values of the ratio of the dissipated acoustic power over the incident acoustic power for the rounded diaphragm are plotted on Figure 3 as a function of frequency for a Mach number $M = 0.0475$ (symbols) and without mean flow (solid lines).

Without mean flow ξ_{max} is about 0.02, while it increases significantly in presence of flow and remains independent of the frequency troughout the investigated range. The variation of ξ_{max} as a function of the Mach number is depicted on Figure 4 for the sharp and rounded diaphragms.

The rather abrupt increase of ξ_{max} with increasing Mach number between $M = 0$ and $M = 0.02$ indicates a change in the nature of the dissipation. Without flow the dissipation is dominated by the viscous effects near the edge of the diaphragm [12]. This is confirmed by the value of ξ_{min} close to zero. There is almost no dissipation when the diaphragm is at an acoustic velocity node of a standing wave patern. In the presence of mean flow, flow separation occurs at the edge of the diaphragm. This allows part of the acoustica energy to be tranferred to vorticity modes. For sharp edges, this exchange of energy can be described in term of an inviscid model based on the Kutta condition applied at the edges (see, for instance, [13] and [14]). The dissipation of the energy of the acoustically generated vorticity takes place in a turbulent mixing zone downstream of the diaphragm. The dissipation process does not interfere strongly with the acoustic plane wave propagation. As in the absence of mean flow, because the acoustic velocity is the driving element of the tranfert from acoustic to vorticity modes, a value $\xi_{\min}$ close to zero is found which correspond to a position of the diaphragm at a velocity node of the acoustic field.

4.2 laminar element in presence of flow

The scattering matrix of a laminar element was measured. This element is a part of a catalytic converter containing parallel open tubes of square section (0.8 mm $\times$ 0.8 mm) separated by 0.2 mm of ceramic material. The length of this element is 200 mm. The values of $\xi_{\min}$ and $\xi_{\max}$ for this element are depicted on Figure 5 as a function of the frequency for a Mach number $M = 0.0605$ and without flow.

The values of $\xi_{\min}$ are almost indentical with and without mean flow while the values of $\xi_{\max}$ is only increased by 10 % to 50 % by the presence of a mean flow. This behaviour shows that the very nature of the dissipative process (visco-thermal effects in the small open tubes) is not affected by the flow. More precisely, a very simple calculation (at low frequency without flow) shows that $\xi_{\min}$ corresponds to the thermal effects that prevail in a pressure anti-node while $\xi_{\max}$ correspond to the viscous effects that prevail in a pressure node. The results on Figure 5 indicates that the thermal dissipation is unaffected by a mean flow while the viscous dissipation is increased by the flow.

For frequencies higher than 400 Hz, $\xi_{\min}$ and $\xi_{\max}$ are not very different. The dissipation of acoustic energy is therefore not influenced by the position of the element in the acoustic system.

5 Conclusion

The acoustic energy conservation law in presence of flow is efficiently described by two thermodynamic variables: acoustic mass velocity and acoustic exergy. The minimal $\xi_{\min}$ and maximal $\xi_{\max}$ ratio of the dissipated or generated a coustic power over the incident acoustic energy can be computed from the eigenvalues of the product of the scattering matrix by its Hermitian transpose. It was demonstrated, in two practical situations (a diaphragm and a laminar element in presence of flow), that some of the physics of the dissipative process can be deduced from the Mach number and the frequency dependences of the potentiality $\xi_{\min}$ and $\xi_{\max}$.

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List of symbols

c_0	sound speed in fluid
E	acoustic energy density
$\mathbf{I}$	acoustic energy flux
$\mathbf{m}'_a$	acoustic mass velocity
m'_x	axial projection of $\mathbf{m}'_a$
p'	fluctuating pressure
$\mathcal{P}$	rate of acoustic energy production per unit of volume
s_0, s'	main and fluctuating entropy
S_i	cross-section of the duct i
$\mathbf{u}_0$, $\mathbf{u}'$	main and fluctuating total velocity
u_0, u'	axial projection of the main and fluctuating total velocity
$\mathbf{v}_0$, $\mathbf{v}'$	irrotational part of the main and fluctuating velocity
W_i	sound power in duct i
$\mathbf{y}$	transverse position vector
Π'	fluctuating external mechanical energy (acoustic exergy)
$\psi_m(\mathbf{y})$	duct normalised eigenfunction: $\int_{S_i} \psi_m \psi_n dS = S_i \, \delta_{mn}$
ρ_0,ρ'	main and fluctuating density
ρ'_a	acoustic part of the fluctuating density ($\rho'_a = p'/c_0^2$)

Figure caption

Figure 1 : *description of the element.*

Figure 2 : *schematic description of the experimental set-up.*

Figure 3 : *minimal and maximal ratio of the dissipated acoustic power over the incident acoustic power for the rounded diaphragm as a function of frequency.* $M = 0.0475$ *:* $\circ$ *,* $M = 0$ *: solid lines.*

Figure 4 : *maximal ratio of the dissipated acoustic power over the incident acoustic power for the rounded diaphragm (*$\circ$*) and for the sharp diaphragm (*$\star$*) as a function of the Mach number (the line is only a fit of the data).*

Figure 5 : *minimal and maximal ratio of the dissipated acoustic power over the incident acoustic power for a laminar element as a function of frequency.* $M = 0.0605$ *:* $\star$ *,* $M = 0$ *: solid lines.*

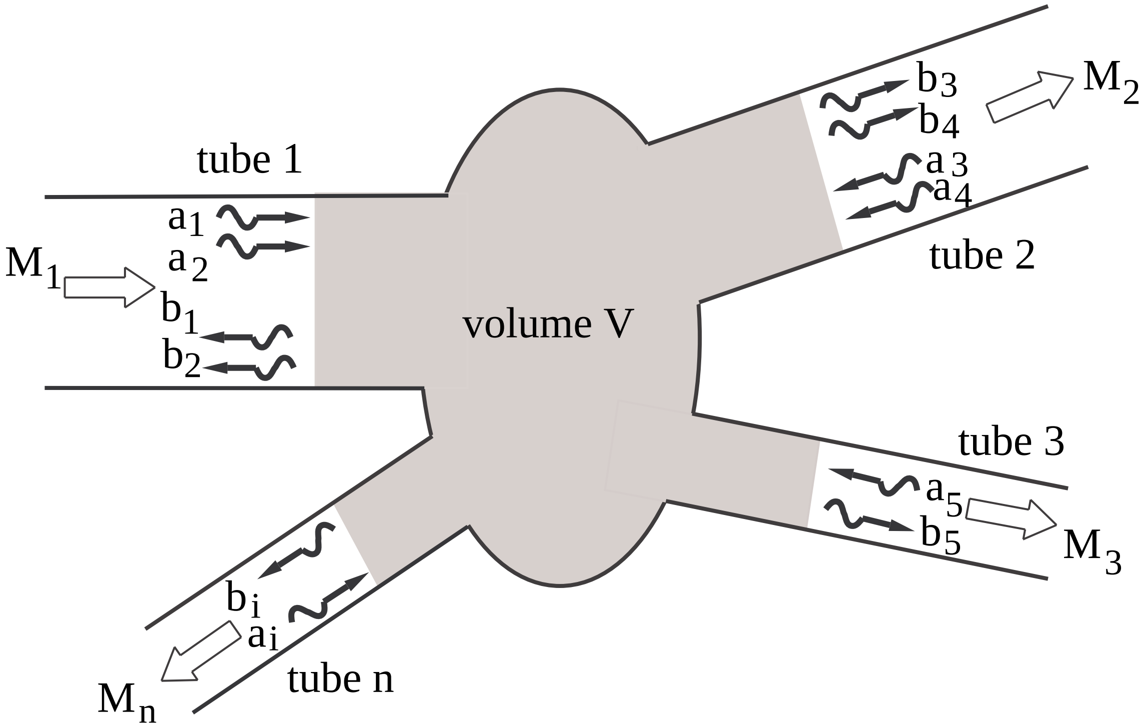

Figure 1: Y. Aurégan R. Starobinski

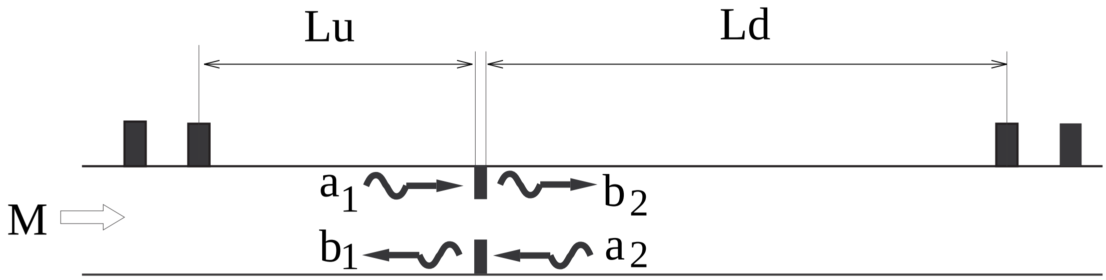

Figure 2: Y. Aurégan R. Starobinski

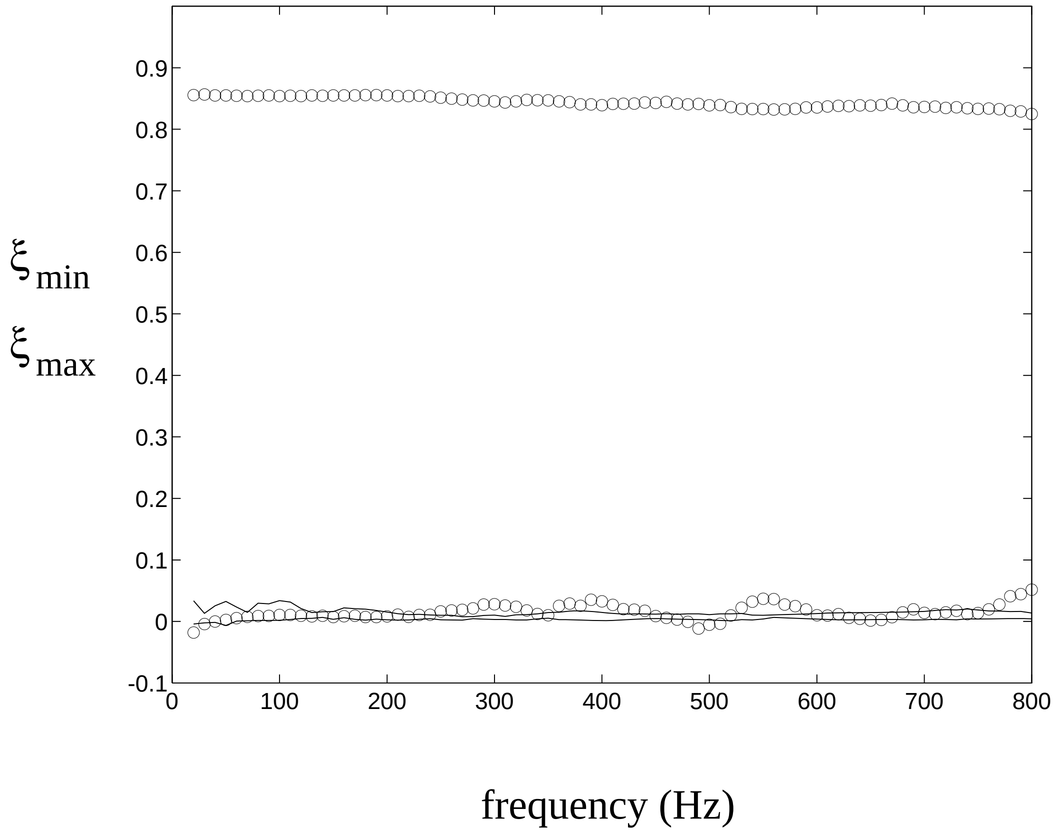

Figure 3: Y. Aurégan R. Starobinski

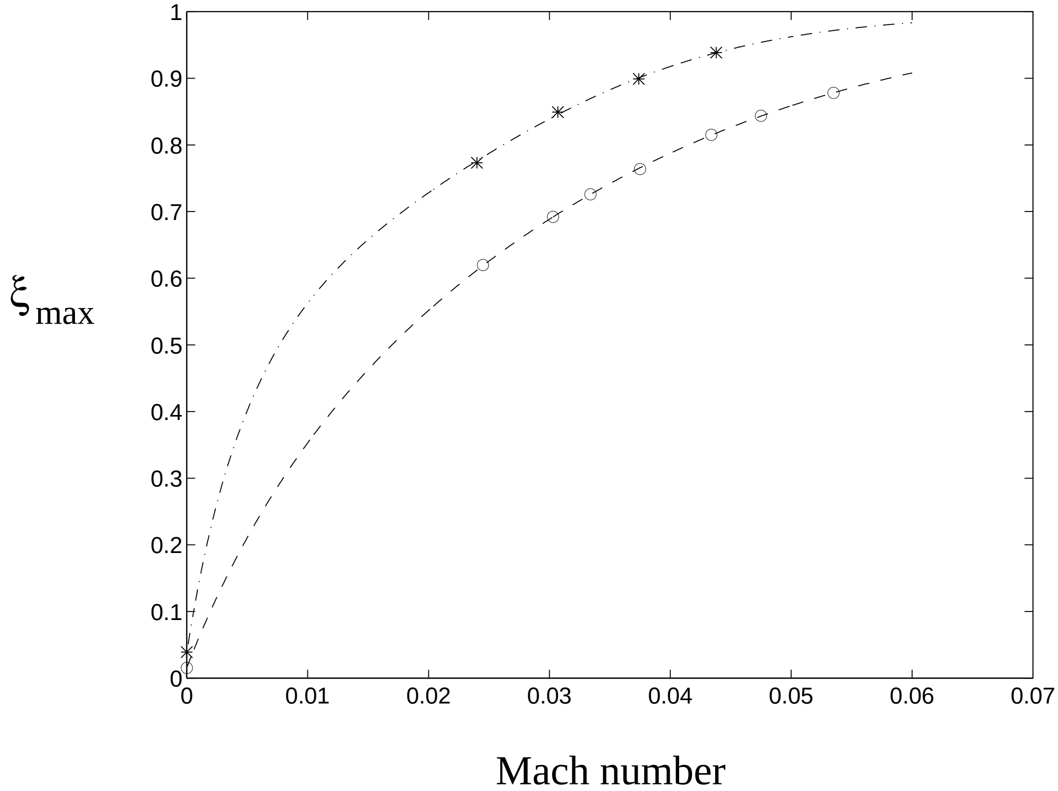

Figure 4: Y. Aurégan R. Starobinski

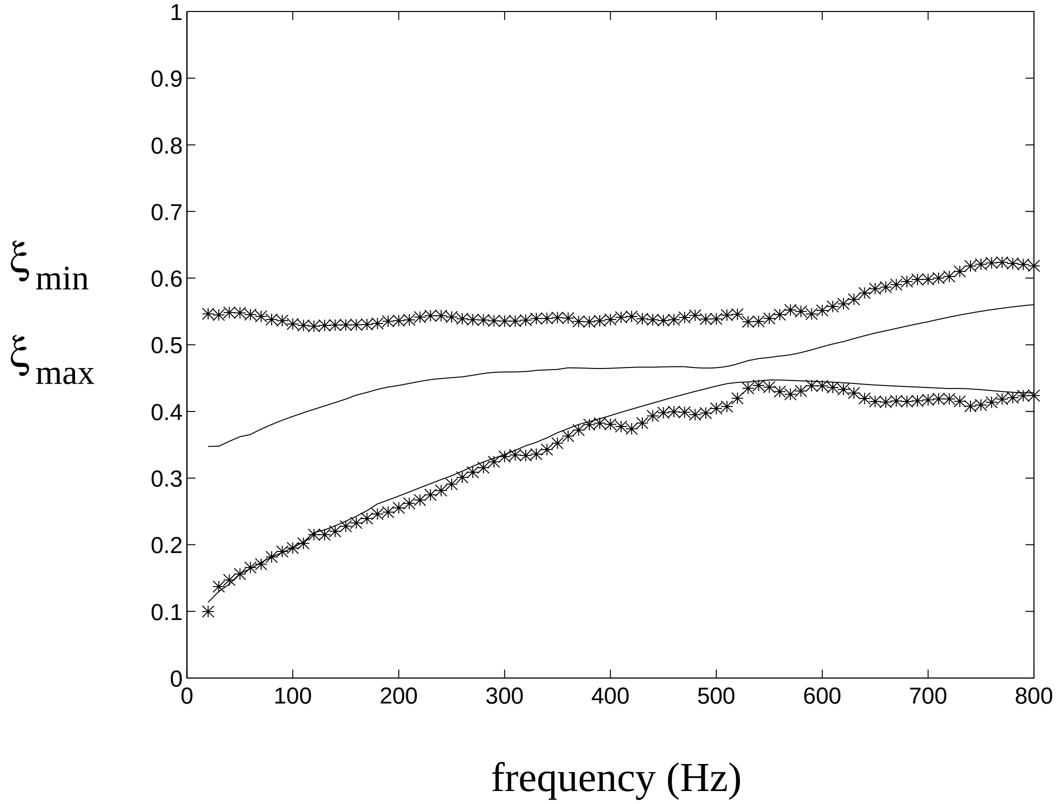

Figure 5: Y. Aurégan R. Starobinski